RUB-TPII-22/98; TPR-98-38

The intrinsic charm contribution to the proton spin

M.V. Polyakov[1,2], A. Schäfer[3], O.V. Teryaev[4]

[1] *Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia.*

[2] *Institut für Theoretische Physik II,*
Universität Bochum, D–44780 Bochum

[3] *Institut für Theoretische Physik*
Universität Regensburg
D-93040 Regensburg, Germany

[4] *Bogoliubov Laboratory of Theoretical Physics*
Joint Institute for Nuclear Research, Dubna
141980 Russia

Abstract

The charm quark contribution to the first moment of $g_1(x, Q^2)$ is calculated using a heavy mass expansion of the divergence of the singlet axial current. It is shown to be small.

The size of a possible intrinsic charm contribution in the proton has been the topic of intensive discussions [1, 2, 3, 4] for many years. It is therefore a natural question to investigate the *polarized* intrinsic charm distribution in the nucleon [5]. Recently one of us and collaborators argued [6] that earlier treatments of the polarized charm contribution to the η and η' [7, 8] were incorrect. In this contribution we extent and adopt that analysis to the nucleon. More precisely we shall focus on the intrinsic charm contribution to the first moment of the spin structure function $g_1(x, Q^2)$. This is known to be intimately related to the gluonic axial anomaly [9, 10, 11, 12]. It may be expressed as forward limit of $G_A^{(0)}(t)$, the form factor in the proton matrix element of the singlet axial current

$$\begin{aligned}\langle N(p_2, \lambda_2)| j_{5\mu}^{(0)}(0) | N(p_1, \lambda_1)\rangle \\ = \bar{u}_N^{(\lambda_2)}(p_2) \left(G_A^{(0)}(t)\gamma_\mu\gamma_5 - G_P^{(0)}(t) q_\mu \gamma_5 \right) u_N^{(\lambda_1)}(p_1),\end{aligned} \tag{1}$$

where $q = p_2 - p_1$ and $t = q^2$. The singlet pseudoscalar form factor does not acquire a Goldstone pole at $t = 0$, even in the chiral limit, contrary to the matrix elements of the octet currents. In this limit, there exist eight massless pseudoscalar mesons serving as Goldstone bosons. However, the ninth pseudoscalar, the η'-meson, remains massive, due to the mixing with the QCD ghost pole.
This fact allows to relate the forward matrix element of the axial current to the (slightly) off-forward one of its divergence:

$$\begin{aligned}\lim_{t\to 0}\langle N(p_2, \lambda_2)| \partial^\mu j_{5\mu}^{(0)}(0) | N(p_1, \lambda_1)\rangle \\ = 2m_N G_A(0) \bar{u}_N^{(\lambda_2)}(p_2)\gamma_5 u_N^{(\lambda_1)}(p_1),\end{aligned} \tag{2}$$

m_N being the proton mass. The divergence of the singlet axial current in turn contains a normal and an anomalous piece,

$$\partial^\mu j_{5\mu}^{(0)} = 2i \sum_q m_q \bar{q}\gamma_5 q - \left(\frac{N_f \alpha_s}{4\pi}\right) G_{\mu\nu}^a \widetilde{G}^{\mu\nu,a}, \tag{3}$$

where N_f is the number of flavours. The two terms at the r.h.s. of the last equation are known to cancel in the limit of infinite quark mass [11, 12, 13]. This is the so-called cancellation of physical and regulator fermions, related to the fact, that the anomaly may be regarded as a usual mass term in the infinite mass limit, up to a sign, resulting from the subtraction in the definition of the regularized operators.
Consequently, one should expect, that the contribution of infinitely heavy quarks to the first moment of g_1 is zero. This is exactly what happens in a perturbative calculation of the triangle anomaly graph [12]. One may wonder, what is the size of this correction for large, but finite masses and how does it compare with the purely perturbative result. To answer this question, one should calculate the r.h.s. of (3) for heavy fermions. The leading coefficient is of the order m^{-2}, and its calculation was addressed recently by two groups [7] and [8] who came up with results differing by a factor of six. However, the operator $f_{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c$ appearing in both treatments, does not satisfy some basic properties, such that both calculations seem to be flawed.

i) It is not a divergence of a local operator, therefore it is not clear that its forward matrix element (2) will vanish.
iii) It makes no contact with the calculation of the triangle diagram in momentum space [14, 12] being essentially non-abelian.

The recent contribution [6] corrected this result and arrived at the expression:

$$\partial^\mu j^c_{5\mu} = \frac{\alpha_s}{48\pi m_c^2}\partial^\mu R_\mu \tag{4}$$

where

$$R_\mu = \partial_\mu \left(G^a_{\rho\nu}\tilde{G}^{\rho\nu,a}\right) - 4\left(D_\alpha G^{\nu\alpha}\right)^a \tilde{G}^a_{\mu\nu}\ . \tag{5}$$

[Here we use the conventions: $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\varepsilon_{0123} = 1$] This result is an explicit 4-divergence and has the Abelian limit. Moreover, this result can also be obtained by a $1/m$ expansion of the triangle diagram contribution. Actually the result of [6] demonstrates that in order m_c^{-2} the entire result (5) can be restored from the venerable triangle diagram. The diagrams with larger number of "legs" give only contributions to the non-abelian part of the result (5). Indeed, computing the forward matrix element of operator (5) between two virtual gluon states we get the following expressions:

$$\langle p|\frac{\alpha_s}{48\pi m_c^2}R_\mu|p\rangle = -i\frac{\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_c^2}\ . \tag{6}$$

On other hands the result of a calculation of the triangle diagram with massive fermions (see e.g. [11]) has a form:

$$\begin{aligned}\langle p|\bar{c}\gamma_\mu\gamma_5 c|p\rangle &= i\frac{\alpha_s}{2\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\Big\{1-\int_0^1 dx\frac{2m_c^2(1-x)}{m_c^2-p^2x(1-x)}\Big\}\\ &= -i\frac{\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_c^2}+O(\frac{1}{m_c^4})\ .\end{aligned} \tag{7}$$

This expression coincides exactly with the result (6). In order to complete the proof it is enough to consider the off-forward matrix element of the operator (4) between two gluons at zero virtuality and compare the result with the expression for the triangle diagram for $\langle p'|\partial^\mu\bar{c}\gamma_\mu\gamma_5 c|p\rangle$. It is easy to check that again the results coincide.
The proton matrix element of R_μ takes a form analogous to that of (1)

$$\begin{aligned}&\langle N(p_2,\lambda_2)|R_\mu(0)|N(p_1,\lambda_1)\rangle\\ &\qquad = \bar{u}_N^{(\lambda_2)}(p_2)\left(G_A^R(t)\gamma_\mu\gamma_5 - G_P^R(t)q_\mu\gamma_5\right)u_N^{(\lambda_1)}(p_1),\end{aligned} \tag{8}$$

It is crucial, that because of the explicit gauge invariance of R_μ the zero mass ghost pole does not contribute. This make an apparent difference with respect to the massless case, when the divergence of the gauge-dependent topological current K_μ appears and the ghost pole contribution does not allow to deduce the relation between the matrix elements of the currents starting from the relation for their divergencies [13]. In the case under investigation only the contribution of the massive η' meson may appear so that

$$\begin{aligned}&\lim_{t\to 0}\langle N(p_2,\lambda_2)|\partial_\mu R_\mu(0)|N(p_1,\lambda_1)\rangle\\ &\qquad = 2m_N G_A^R(0)\bar{u}_N^{(\lambda_2)}(p_2)\gamma_5 u_N^{(\lambda_1)}(p_1),\end{aligned} \tag{9}$$

The contribution of charm to the forward matrix element can be obtained by substituting (1, 8) into the proton matrix elements of (4), giving in the forward limit.

$$\langle N(p,\lambda)|j_{5\mu}^{(c)}(0)|N(p,\lambda)\rangle = \frac{\alpha_s}{48\pi m_c^2}\langle N(p,\lambda)|R_\mu(0)|N(p,\lambda)\rangle \tag{10}$$

In deriving this expression we used (2, 9). Note that the first term in R_μ does not contribute to the forward matrix element because of its gradient form, while the contribution of the second one is rewritten, by making use of the equation of motion, as matrix element of the operator

$$\begin{aligned}\langle N(p,\lambda)|j_{5\mu}^{(c)}(0)|N(p,\lambda)\rangle &= \tfrac{\alpha_s}{12\pi m_c^2}\langle N(p,\lambda)|g\sum\nolimits_{\mathrm{f=u,d,s}}\bar\psi_f\gamma_\nu\tilde G_\mu{}^\nu\psi_f|N(p,\lambda)\rangle \\ &\equiv \tfrac{\alpha_s}{12\pi m_c^2}2m_N^3 s_\mu f_S^{(2)},\end{aligned} \tag{11}$$

The parameter $f_S^{(2)}$ was determined before in calculations of the power corrections to the first moment of the singlet part of g_1 part of which is given by exactly the quark-gluon-quark matrix element we got. Note that within our $1/m_c$ approximation the c contribution to the flavour sum can be neglected. QCD-sum rule calculations gave $f_S^{(2)} = \frac{9}{5}(f^{(2)}(\mathrm{proton}) + f^{(2)}(\mathrm{neutron})) = 0.09$[1] [15], estimates using the renormalon approach led to $f_S^{(2)} = \pm 0.02$ [16] and calculations in the instanton model of the QCD vacuum give a result very close to that of QCD sum rule [15] [17].
Inserting these numbers we get finally for the charm axial constant the estimate

$$\bar G_A^c(0) = -\frac{\alpha_s}{12\pi}f_S^{(2)}(\frac{m_N}{m_c})^2 \approx -5\cdot 10^{-4} \tag{12}$$

with probably a 100 percent uncertainty (see e.g. [18]). As the mass term in the triangle diagram is coming from the region of transverse momenta of the order m_c, this should be the correct scale of both α_s and $f_S^{(2)}$. Because this scale is not far from the typical hadronic scale at which $f_S^{(2)}$ was estimated we can neglect evolution effects.
Note that this contribution is of non-perturbative origin (therefore we call it intrinsic), so that it is sensitive to large distances, as soon as the factorization scale is larger than m_c. If the scale is also larger than m_b, one can immediately conclude that the non-perturbative bottom contributions is further suppressed by the factor $(m_c/m_b)^2 \sim 0.1$.
Let us note, that the naive application of our approach to the case of strange quarks gives for their contribution to the first moment of g_1 roughly $-5\cdot 10^{-2}$, which is compatible with the experimental data. The possible applicability of a heavy quark expansions for strange quarks in a similar problem was discussed earlier [19] in the case of the vacuum condensates of heavy quarks. That analyses was also related to the anomaly equation for heavy quarks, however, for the trace anomaly, rather than the axial one.

Let us summarize: We have related the non-perturbative contribution of charm quarks to the nucleon spin (at scale m_c) to the singlet twist-4 coeffcent appearing e.g. in the Ellis-Jaffe sum rule. Numerically it is found to be very small, contrary to the suggestion of [5, 8]. We would like to note that in a recent paper [20] it was shown that

[1] Note that here we use a convention for the ε-tensor which differs by sign from that of [15]

also the perturbative Δc contribution is very small. We see this as further support for our result.

Acknowledgement:
We thank K. Goeke, M. Franz, M. Maul, S.V. Mikhailov, P.V. Pobylitsa and E. Stein for helpful discussions. O.V.T. was supported by the Russian Foundation for the Basic Research (grant 96-02-17631), the Graduiertenkolleg Erlangen-Regensburg (DFG) and Bochum University. M.V.P. is grateful to Klaus Goeke for encouragement and kind hospitality at Bochum University. The work of M.V.P. has been supported in parts by a joint grant of the Russian Foundation for Basic Research (RFBR) and the Deutsche Forschungsgemeinschaft (DFG) 436 RUS 113/181/0 (R), by the BMBF grant RUS-658-97 and the COSY (Jülich). A.S. acknowledges support from BMBF.

References

[1] S.J. Brodsky, P. Hoyer, C. Peterson and N. Sakai, Phys. Lett. B93 (1980) 451
S.J. Brodsky and C. Peterson, Phys. Rev. D23 (1981) 2745

[2] E. Hoffmann and R. Moore, Z.Phys. C20 (1983) 71

[3] B.W. Harris, J. Smith and R. Vogt, Nucl. Phys. B461 (1996) 181

[4] G. Ingelman and M. Thunman, Z. Phys. C73 (1997) 505

[5] I. Halperin and A. Zhitnitsky, 'Polarized intrinsic charm as a possible solution to the proton spin problem', hep-ph/9706251; A.Blotz and E. Shuryak 'Instanton-induced charm contribution to polarized deep-inelastic scattering', hep-ph/9710544.

[6] M. Franz, P.V. Pobylitsa, M.V. Polyakov and K. Goeke, 'On the heavy quark mass expansion for the operator $\bar{Q}\gamma_5 Q$ and the charm content of η, η', hep-ph/9810343 preprint

[7] I. Halperin and A. Zhitnitsky, Phys. Rev. D56 (1997) 7247

[8] F. Araki, M. Musakhanov and H. Toki, 'Axial currents of virtual charm in light quark processes', hep-ph/9808290 preprint

[9] A.V. Efremov and O.V. Teryaev, Report JINR-E2-88-287, Czech.Hadron Symp.1988, p.302.

[10] G. Altarelli and G.G. Ross, Phys. Lett. B212 (1988) 391.

[11] R.D. Carlitz, J.C. Collins and A.H. Mueller, Phys. Lett. B214 (1988) 229

[12] L. Mankiewicz and A. Schäfer, Phys. Lett. B242 (1990) 455

[13] A.V. Efremov, J. Soffer and O.V. Teryaev, Nucl.Phys. B346 (1990) 97

[14] S.L. Adler, Phys.Rev. 177 (1969) 2426

[15] E. Stein, P. Gornicki, L. Mankiewicz and A. Schäfer, Phys. Lett. B353 (1995) 107

[16] E. Stein, M. Maul, L. Mankiewicz, and A. Schäfer, 'Renormalon model predictions for power-corrections to flavour singlet deep inelastic structure functions', hep-ph/9803342 preprint

[17] J. Balla, M.V. Polyakov and C. Weiss, Nucl. Phys. B510 (1998) 327

[18] B.L. Ioffe, Phys. Atom. Nucl. **60** (1997) 1707.

[19] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl.Phys. B147 (1979) 385 (section 6.8); D.J. Broadhurst and S.C. Generalis, Phys. Lett. B139 (1984) 85.

[20] J. Bluemlein and W. L. van Neerven, 'Heavy flavor contribution to the deep inelastic scatterint sum rules', hep-ph/9811351